



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2026

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ( $g$ ) is needed, use  $10 \text{ m s}^{-2}$ .

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.



- 1** A particle  $P$  of mass  $m$  is moving in a horizontal circle with constant angular speed  $\omega$  on the smooth inner surface of a hemispherical shell of radius  $a$ . The plane of the circular motion is at a height  $h = \frac{1}{3}a$  above the lowest point on the shell. The angle between the vertical and the normal reaction of the surface on  $P$  is  $\theta$ .

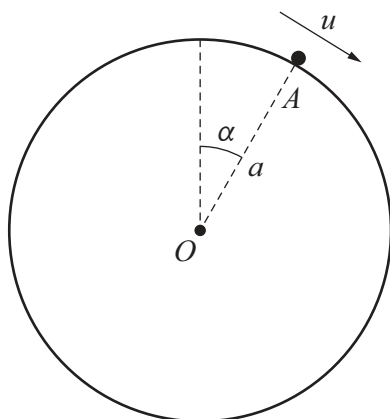
Find  $\omega$  in terms of  $a$  and  $g$ .

[5]

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- [6]



A fixed smooth solid sphere has centre  $O$  and radius  $a$ . A particle of mass  $m$  is projected perpendicular to  $OA$  in a downward direction with speed  $u$  from the point  $A$  on the outer surface of the sphere, where  $OA$  makes an angle  $\alpha$  with the upward vertical through  $O$  (see diagram).

The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point  $B$ , where  $OB$  makes an angle  $\beta$  with the upward vertical through  $O$ . The speed of the particle at the point  $B$  is  $v$ .

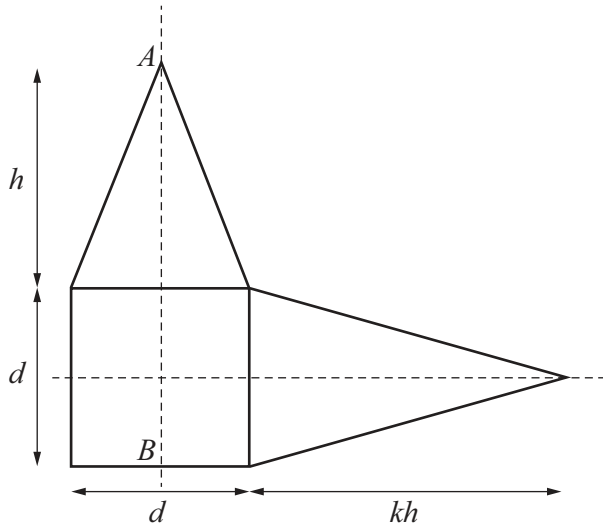
Given that the ratio of  $u$  to  $v$  is  $2 : 3$  find the ratio  $\cos \alpha : \cos \beta$ . [5]

[illegible]



5

[illegible]



An object consists of a uniform solid cube of side length  $d$ , and two uniform solid square-based pyramids. The cube and the pyramids are made of the same material.

One pyramid, of base length  $d$  and height  $h$ , is fixed to the top face of the cube so that the base of this pyramid and the face of the cube coincide. Point  $A$  is at the top vertex of this pyramid. Point  $B$  is in the centre of the bottom face of the cube, vertically below  $A$ . The second pyramid, of base length  $d$  and height  $kh$ , is fixed to a vertical face of the cube so that the base of this pyramid and the vertical face of the cube coincide (see diagram, which shows a cross-section, including the axes of symmetry).

- (a) Find an expression for the distance of the centre of mass of the object from  $AB$  in terms of  $h$ ,  $k$ , and  $d$ .

[4]

[illegible]



The object is placed on a horizontal plane so that the bottom face of the cube that contains point  $B$  is in contact with the plane. It is given that the object is on the point of toppling.

- (b) Find the value of  $h$  in terms of  $k$  and  $d$ . [3]

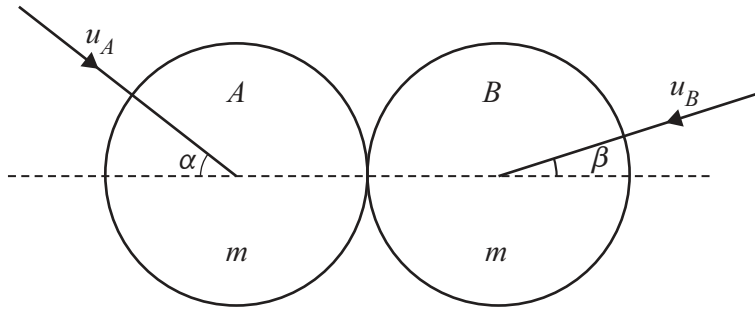
[illegible]

- (c) Given that  $h = 1.5d$ , find the value of  $k$ . [2]

This image shows a single sheet of white paper with ten evenly spaced horizontal dotted lines, typical of primary school writing paper. The lines are light gray and extend across the full width of the page. There is no handwriting or other markings on the paper.



5



Two uniform smooth spheres  $A$  and  $B$  of equal radii and equal masses  $m$  are moving with speeds  $u_A$  and  $u_B$  respectively on a smooth horizontal surface when they collide. Immediately before the collision, the direction of motion of  $A$  makes an angle  $\alpha$  with the line of centres, and the direction of motion of  $B$  makes an angle  $\beta$  with the line of centres (see diagram). Immediately after the collision, the direction of motion of  $A$  makes an angle  $\theta$  with the line of centres, and the direction of motion of  $B$  makes an angle  $\varphi$  with the line of centres. It is given that  $\cos \alpha = \cos \varphi = \frac{4}{5}$  and  $\cos \beta = \frac{24}{25}$  and  $\cos \theta = \frac{3}{5}$ .

- (a) Find the percentage loss in the kinetic energy of  $A$ .

[2]

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**(b)** Find the value of the coefficient of restitution between the spheres.

[4]

[illegible]

- 6 A smooth plane is inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{4}{5}$ . One end of a light elastic spring, with natural length  $10a$  and modulus of elasticity  $1.6mg$ , is attached to a fixed point  $O$  on the plane. The other end of the spring is attached to a particle  $P$  of mass  $0.2m$ . The point  $O$  and the particle  $P$  lie on the same line of greatest slope of the plane, with  $P$  above  $O$ . The particle  $P$  is at rest.

(a) Show that the distance  $OP$  is  $9a$ .

[3]

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A particle  $Q$  of mass  $0.3m$  is held on the plane on the line of greatest slope through  $O$ , at a distance  $10a$  above  $P$ . The particle  $Q$  is released from rest. The particle  $Q$  then collides and coalesces with particle  $P$ .

(b) Find, in terms of  $a$  and  $g$ , the speed of the combined particle immediately after the collision. [4]

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[illegible]



- 7 At time  $t = 0$  s, a particle  $P$  is projected from a point  $O$  on horizontal ground. The initial velocity of  $P$  is  $25 \text{ ms}^{-1}$  at an angle  $\alpha$  above the horizontal, where  $\sin \alpha = \frac{4}{5}$ . At time  $t = T$  s, a second particle  $Q$  is projected vertically upwards from the horizontal ground directly below the point where  $P$  will reach its greatest height. The initial velocity of  $Q$  is also  $25 \text{ ms}^{-1}$ . Both particles move freely under gravity. The two particles collide when  $Q$  is travelling upwards and at the point where  $P$  reaches its greatest height.

(a) Show that the two particles collide 2 seconds after  $P$  is projected. [1]

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(b) Find the value of  $T$ . [3]

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- (c) As a result of the collision, the trajectory of  $P$  changes, but the horizontal component of its velocity remains unchanged. Immediately after the collision, the speed of  $P$  is  $v \text{ ms}^{-1}$ , at an angle  $\theta$  above the horizontal.

Find, in terms of  $\tan \theta$ , the greatest height of  $P$  above the ground after the collision. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]





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